

# **Optimal Broadcasting Strategies for Conjunctive Queries over Distributed Data**

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# Outline

1. Setting and Context
2. Oblivious Broadcasting Functions
3. Correctness & Optimality
4. Broadcast Dependency Sets
5. Conclusion & Future Work

# Context

CALM conjecture: “Monotonic = No-coordination”

[Hellerstein, 2010]

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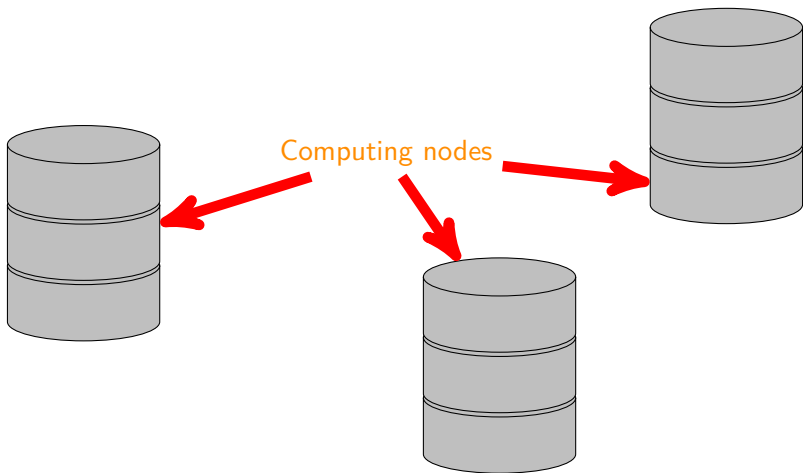
[Ameloot, Neven, Van den Bussche, 2011]

- ▶ Generalization

[Ameloot, Neven, K., Zinn, 2014]

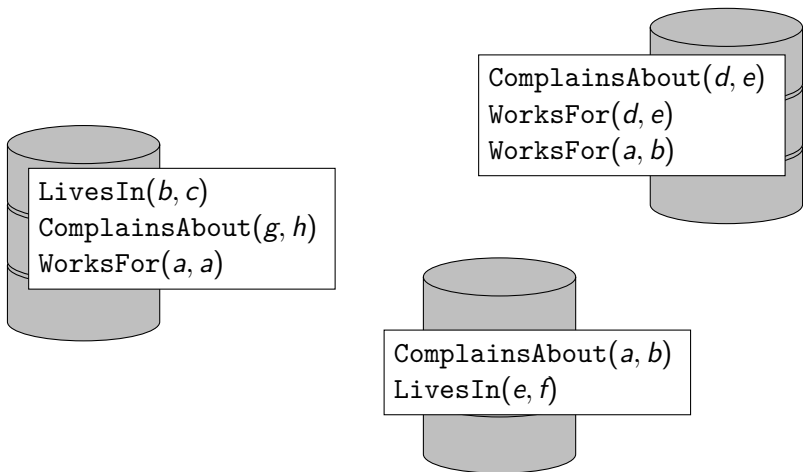
# Setting

A *network*  $\mathcal{N}$  is a set of *computing nodes*.



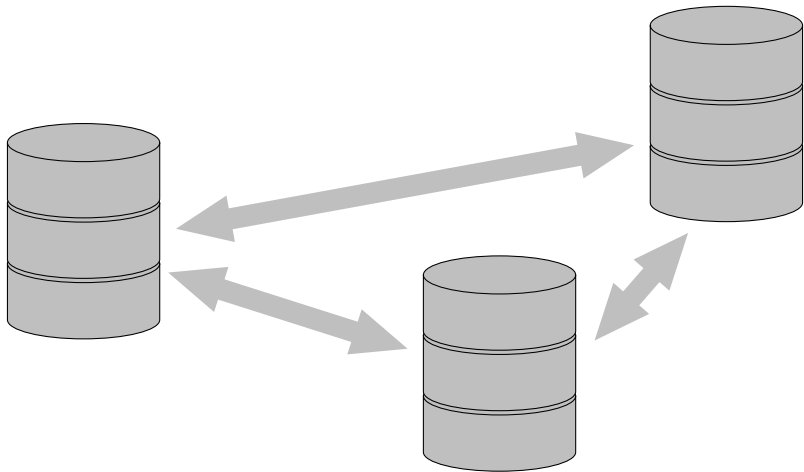
# Setting

A **distribution** is a mapping from nodes onto instances.



# Setting

Communication: Asynchronous

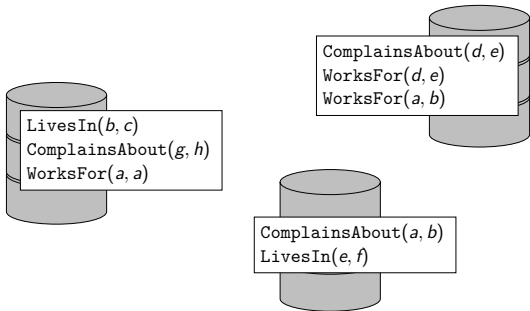




# Monotonic $\subseteq$ No-coordination

Running Example:

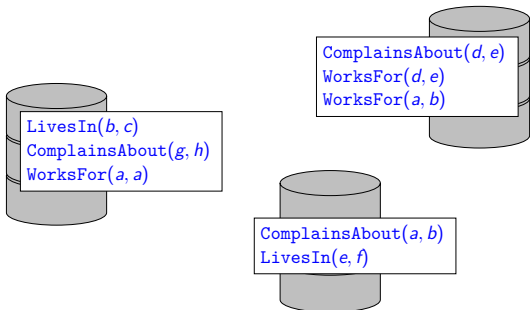
$Q(x, y, z) \leftarrow \text{ComplainsAbout}(x, y), \text{WorksFor}(x, y), \text{LivesIn}(y, z)$



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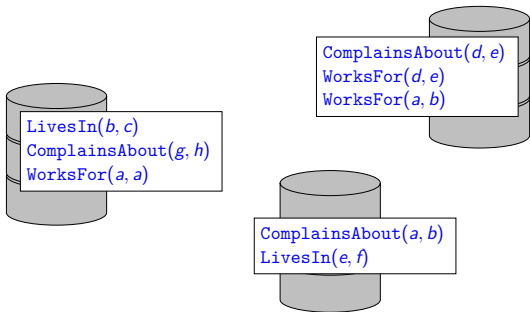


- ▶ Let every node broadcast all of its data;

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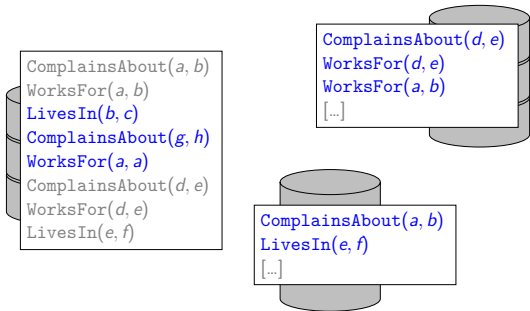


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- ▶ Periodically run  $Q$  locally on every node

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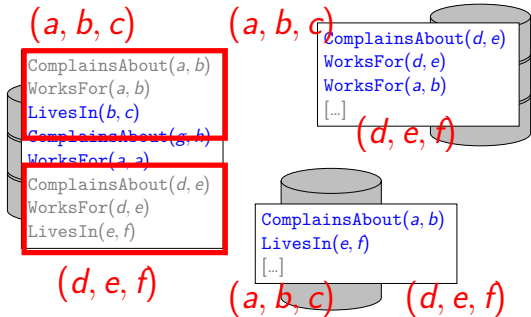


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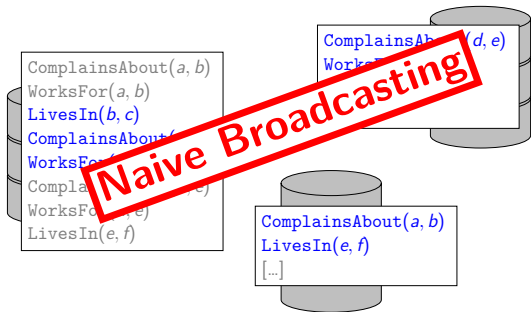


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# Current Work

No-coordination + Broadcast all



No-coordination + Selective broadcasting

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## Full CQs without self-joins

1.  $Q(x, y, z) \leftarrow R(x, y), S(y, z)$
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3.  $Q(x, y) \leftarrow R(x, y), R(y, x)$



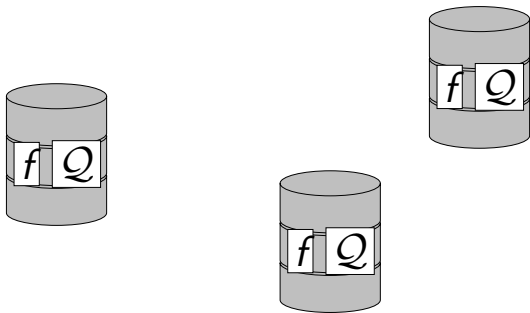
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# Oblivious Broadcasting Functions

## Definition

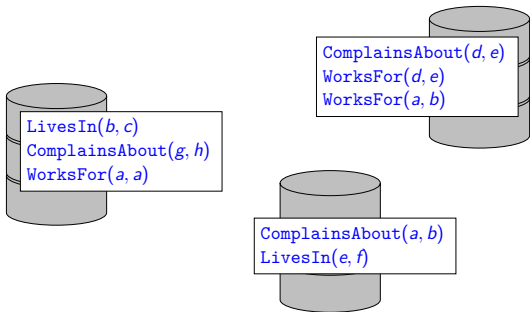
Let  $f$  be a total function from instances to instances. We call  $f$  an **oblivious broadcasting function (OBF)** if  $f$  is generic, and  $f(I) \subseteq I$  for every instance  $I$ .



# Running Example: Naive Broadcasting

$Q(x, y, z) \leftarrow \text{ComplainsAbout}(x, y), \text{WorksFor}(x, y), \text{LivesIn}(y, z)$

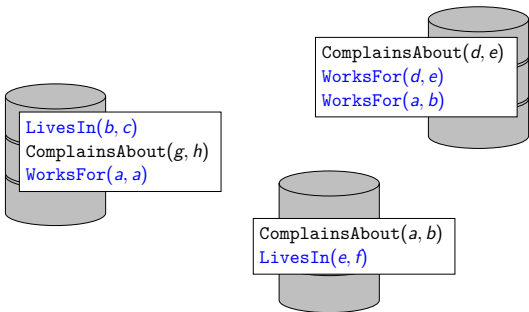
OBF: broadcast everything



## Running Example: Relation-Based

$Q(x, y, z) \leftarrow \text{ComplainsAbout}(x, y), \text{WorksFor}(x, y), \text{LivesIn}(y, z)$

OBF: don't broadcast **ComplainsAbout**( $x, y$ )



## Running Example: Relation-Based

$Q(x, y, z) \leftarrow \text{ComplainsAbout}(x, y), \text{WorksFor}(x, y), \text{LivesIn}(y, z)$

OBF: don't broadcast **ComplainsAbout**( $x, y$ )

WorksFor( $a, b$ )  
LivesIn( $b, c$ )  
ComplainsAbout( $g, h$ )  
WorksFor( $a, a$ )  
WorksFor( $d, e$ )  
LivesIn( $e, f$ )

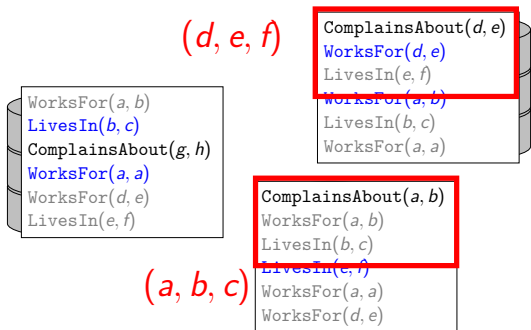
ComplainsAbout( $d, e$ )  
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# Oblivious Broadcasting Functions

Let  $\mathcal{N}$  be a network,  $I$  an instance,  $H$  a distribution of  $I$  over  $\mathcal{N}$ .

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## Broadcast Facts

$$B(f, H) \stackrel{\text{def}}{=} \bigcup_{c \in \mathcal{N}} f(H(c)).$$

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## Broadcast Facts

$$B(f, H) \stackrel{\text{def}}{=} \bigcup_{c \in \mathcal{N}} f(H(c)).$$

## Distributed Output

$$\text{eval}(\mathcal{Q}, f, H) \stackrel{\text{def}}{=} \bigcup_{c \in \mathcal{N}} \mathcal{Q}(H(c) \cup B(f, H))$$



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# Correctness

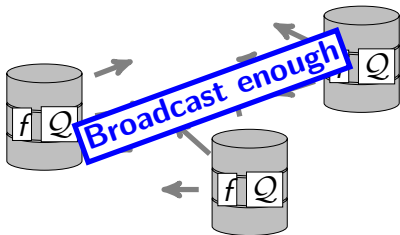
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An OBF is **correct for** CQ  $Q$  if  $Q(I) = eval(Q, f, H)$  for every instance  $I$  and distribution  $H$  for  $I$ .

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## Definition

Let  $\mathcal{Q}$  be a CQ,  $f$  and  $g$  be two distinct facts. We say that  $f$  and  $g$  are **compatible**, written  $f \sim_{\mathcal{Q}} g$ , if there is a valuation  $V$  for  $\mathcal{Q}$  that requires them both.

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## Example

$Q(x, y, z) \leftarrow \text{ComplainsAbout}(x, y), \text{WorksFor}(x, y), \text{LivesIn}(y, z)$

- ▶  $\text{ComplainsAbout}(a, b) \sim_Q \text{LivesIn}(b, c)$
- ▶  $\text{ComplainsAbout}(a, b) \not\sim_Q \text{LivesIn}(a, c)$

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## Lemma

Let  $\mathcal{Q}$  be a CQ and  $f$  be an OBF. Then, the following are equivalent:

1.  $f$  is correct for  $\mathcal{Q}$ ; and
2. there are no instances  $I, J$ , and facts  $f, g$ , with  $f \sim_{\mathcal{Q}} g$ ,  $g \notin I$ ,  $f \notin J$  such that  $f \notin f(I \cup \{f\})$  and  $g \notin f(J \cup \{g\})$ .

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Ideally: “One OBF that is always at least as good as all others”

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$$\|B(f, H)\| \stackrel{\text{def}}{=} \sum_{c \in \mathcal{N}} |f(H(c))|$$

## Definition

An OBF  $f$  for a CQ  $\mathcal{Q}$  is **optimal** if  $\|B(f, H)\| \leq \|B(g, H)\|$  for every other OBF  $g$  for  $\mathcal{Q}$  and for every instance  $I$  and distribution  $H$ .



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**No such OBF exists**

## Proof: No Optimal OBF exists

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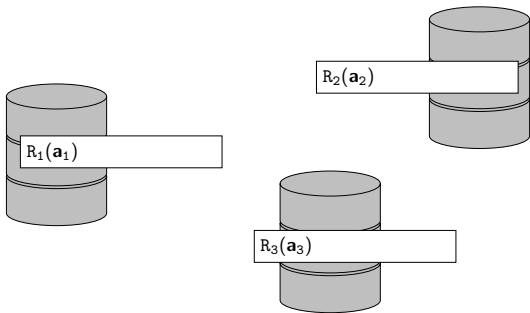
$R_2(\mathbf{a}_2)$

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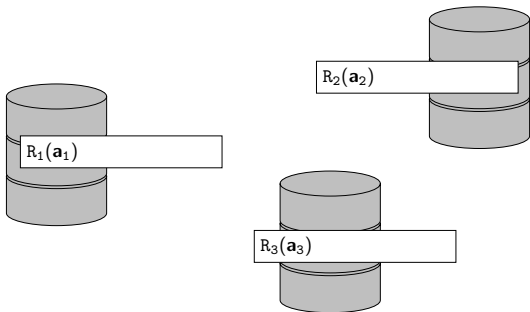
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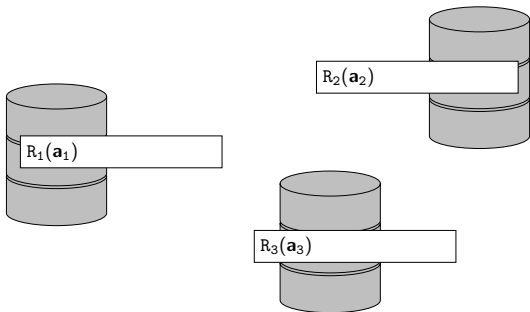
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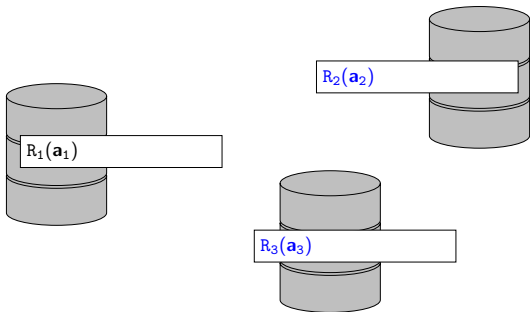
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- ▶ At least two of these facts **must** be broadcast
- ▶ OBFs exist that broadcast **only** two of them

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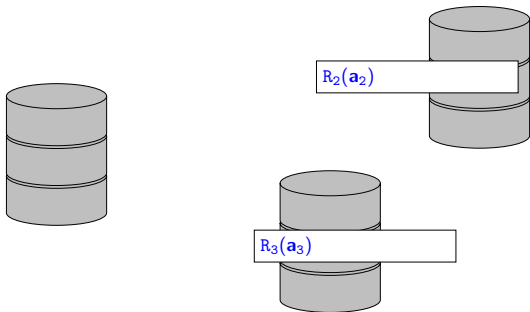


W.l.o.g: OBF does not broadcast  $R_1(\mathbf{a}_1)$



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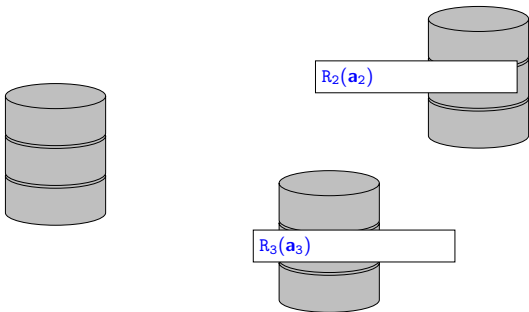
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OBF exists that broadcasts less

# Locally-optimal OBFs

Let  $f$  and  $g$  be OBFs.

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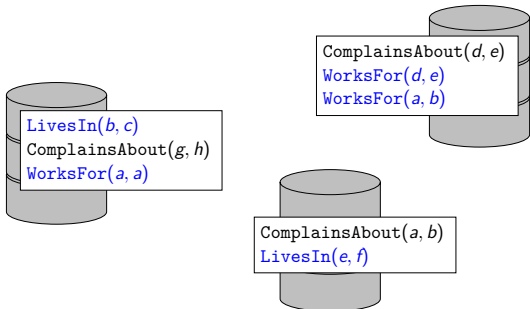
## Definition

An OBF  $f$  that is correct for a CQ  $Q$  is **locally optimal** if for every other OBF  $g$  that is correct for  $Q$ ,  $g \subseteq f$  implies  $f = g$ .

## Running Example: Relation-Based

$Q(x, y, z) \leftarrow \text{ComplainsAbout}(x, y), \text{WorksFor}(x, y), \text{LivesIn}(y, z)$

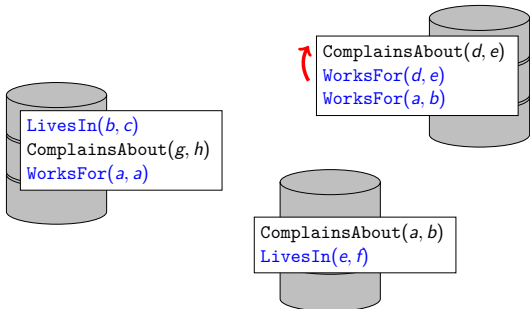
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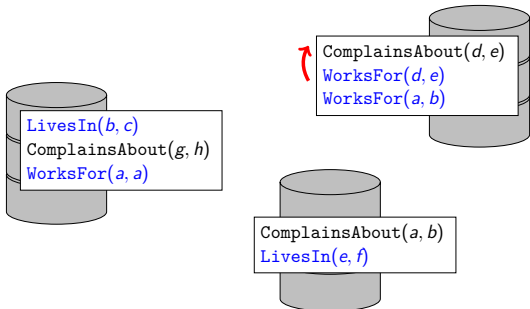


- `WorksFor(d, e)` requires `ComplainsAbout(d, e)`

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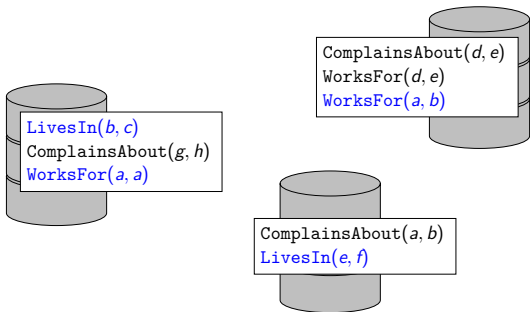


- ▶ `WorksFor(d, e)` requires `ComplainsAbout(d, e)`
- ▶ valuations requiring `ComplainsAbout(d, e)` satisfy locally

## Running Example: Relation-Based

$Q(x, y, z) \leftarrow \text{ComplainsAbout}(x, y), \text{WorksFor}(x, y), \text{LivesIn}(y, z)$

OBF: Don't broadcast **ComplainsAbout**( $x, y$ ) + don't broadcast **WorksFor**( $x, y$ ) if **ComplainsAbout**( $x, y$ ) is present



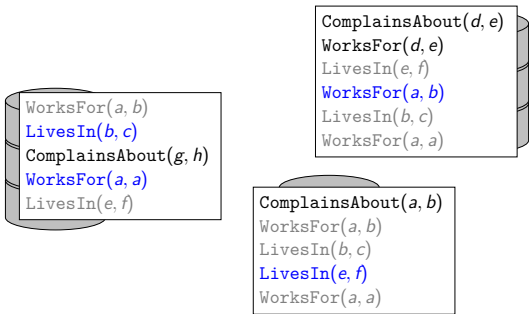
Not necessary to broadcast **WorksFor**( $d, e$ )



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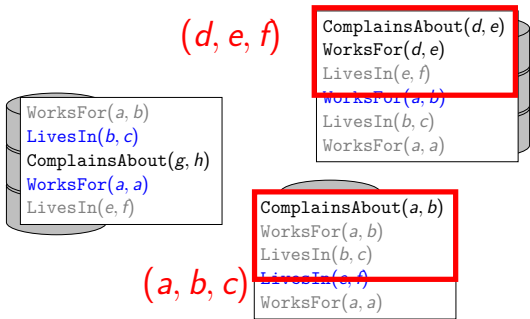
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# Locally-optimal OBFs

## Lemma

Let  $\mathcal{Q}$  be a CQ and let  $f$  be an OBF for  $\mathcal{Q}$ . The following are equivalent:

1.  $f$  is locally optimal; and
2. for every instance  $I$  and fact  $\mathbf{f}$  for which  $\mathbf{f} \in f(I \cup \{\mathbf{f}\})$ , there is an instance  $J$  and a fact  $\mathbf{g}$  such that  $\mathbf{f} \sim_{\mathcal{Q}} \mathbf{g}$ ,  $\mathbf{g} \notin I$ ,  $\mathbf{f} \notin J$ , and  $\mathbf{g} \in f(J \cup \{\mathbf{g}\})$ .

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# Broadcast Dependency Sets

Building blocks: Equality types

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## Example

- ▶ `WorksFor( $x, y$ ),  $x \neq y$`
- ▶ `ComplainsAbout( $x, y$ ),  $x = y$`

# Broadcast Dependency Sets

Building blocks: Equality types

A **Broadcast Dependency set** is a set of tuples  $(\tau, T)$ , where

- ▶  $\tau$  is an equality type consistent with atom of  $\mathcal{Q}$  (**key**)
- ▶  $T$  is a set of equality types consistent with atoms of  $\mathcal{Q}$  (**dependency set**)
- ▶ + additional restrictions

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**Semantics:** Broadcast a fact only if

- ▶ it has a consistent equality type; and
- ▶ either
  - ▶ it does not correspond to a key in the BDS; or
  - ▶ the facts represented by the corresponding dependency set are not all present.



# Complexity Results

**Theorem:** Deciding whether BDS is correct for  $\mathcal{Q}$  is coNP-complete

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**Theorem:** Complete characterization for locally optimal, correct OBFs.

# OBF Construction

Parameter: sequence  $\mathcal{S}$  of all consistent equality-types for  $\mathcal{Q}$ .

- ▶  $D \stackrel{\text{def}}{=} \emptyset$
- ▶ Consume types  $\tau \in \mathcal{S}$  one-by-one:
  - ▶  $values \stackrel{\text{def}}{=} \emptyset$
  - ▶ For every key  $\tau'$  in  $D$  compatible with  $\tau$ , check condition and add to  $values$
  - ▶ On failure: ignore  $\tau$  and jump to the next type
  - ▶ On success: add  $(\tau, values)$  to  $D$ .

Output:  $D$

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**Theorem:** In general: exponential in  $\mathcal{Q}$

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Parameter: sequence  $\mathcal{S}$  of all consistent equality-types for  $\mathcal{Q}$ .

- ▶  $D \stackrel{\text{def}}{=} \emptyset$
- ▶ Consume types  $\tau \in \mathcal{S}$  one-by-one:
  - ▶  $values \stackrel{\text{def}}{=} \emptyset$
  - ▶ For every key  $\tau'$  in  $D$  compatible with  $\tau$ , check condition and add to  $values$
  - ▶ On failure: ignore  $\tau$  and jump to the next type
  - ▶ On success: add  $(\tau, values)$  to  $D$ .

Output:  $D$

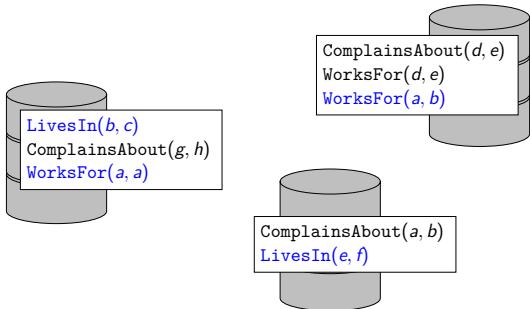
**Theorem:** In general: exponential in  $\mathcal{Q}$

**Theorem:** polynomial in  $\mathcal{Q}$  if only considering relations

## Running Example: Relation-Based

$Q(x, y, z) \leftarrow \text{ComplainsAbout}(x, y), \text{WorksFor}(x, y), \text{LivesIn}(y, z)$

OBF: Don't broadcast **ComplainsAbout**( $x, y$ ) + don't broadcast **WorksFor**( $x, y$ ) if **ComplainsAbout**( $x, y$ ) is present



## Running Example: Partial Atomic Types

$Q(x, y, z) \leftarrow \text{ComplainsAbout}(x, y), \text{WorksFor}(x, y), \text{LivesIn}(y, z)$

OBF:

- ▶ don't broadcast **ComplainsAbout**( $x, y$ )



## Running Example: Partial Atomic Types

$Q(x, y, z) \leftarrow \text{ComplainsAbout}(x, y), \text{WorksFor}(x, y), \text{LivesIn}(y, z)$

OBF:

- ▶ don't broadcast **ComplainsAbout**( $x, y$ ),  $x \neq y$

## Running Example: Partial Atomic Types

$Q(x, y, z) \leftarrow \text{ComplainsAbout}(x, y), \text{WorksFor}(x, y), \text{LivesIn}(y, z)$

OBF:

- ▶ don't broadcast **ComplainsAbout**( $x, y$ ),  $x \neq y$
- ▶ don't broadcast **WorksFor**( $x, y$ ),  $x = y$

## Running Example: Partial Atomic Types

$Q(x, y, z) \leftarrow \text{ComplainsAbout}(x, y), \text{WorksFor}(x, y), \text{LivesIn}(y, z)$

OBF:

- ▶ don't broadcast **ComplainsAbout**( $x, y$ ),  $x \neq y$
- ▶ don't broadcast **WorksFor**( $x, y$ ),  $x = y$
- ▶ don't broadcast **ComplainsAbout**( $x, y$ ),  $x = y$  if **WorksFor**( $x, y$ ) is present
- ▶ don't broadcast **WorksFor**( $x, y$ ),  $x \neq y$  if **ComplainsAbout**( $x, y$ ) is present

## Running Example: Partial Atomic Types

$Q(x, y, z) \leftarrow \text{ComplainsAbout}(x, y), \text{WorksFor}(x, y), \text{LivesIn}(y, z)$

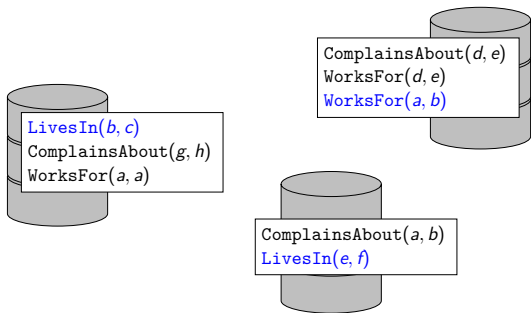
OBF:

- ▶ don't broadcast **ComplainsAbout**( $x, y$ ),  $x \neq y$
- ▶ don't broadcast **WorksFor**( $x, y$ ),  $x = y$
- ▶ don't broadcast **ComplainsAbout**( $x, y$ ),  $x = y$  if **WorksFor**( $x, y$ ) is present
- ▶ don't broadcast **WorksFor**( $x, y$ ),  $x \neq y$  if **ComplainsAbout**( $x, y$ ) is present

Correct + locally optimal

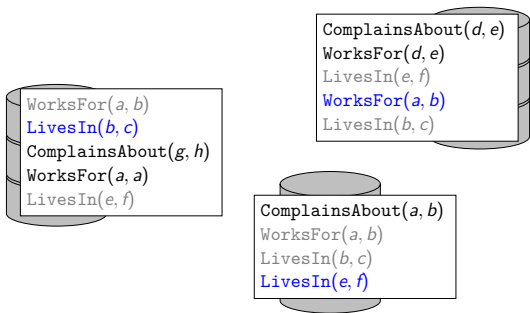
# Running Example: Partial Atomic Types

$Q(x, y, z) \leftarrow \text{ComplainsAbout}(x, y), \text{WorksFor}(x, y), \text{LivesIn}(y, z)$



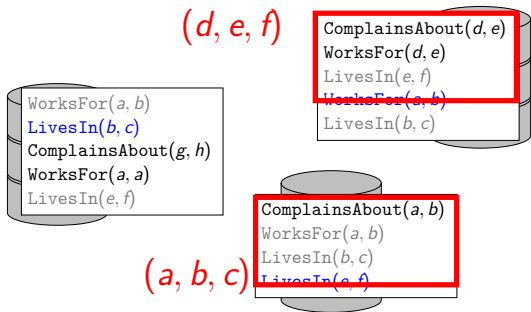
# Running Example: Partial Atomic Types

$Q(x, y, z) \leftarrow \text{ComplainsAbout}(x, y), \text{WorksFor}(x, y), \text{LivesIn}(y, z)$



# Running Example: Partial Atomic Types

$Q(x, y, z) \leftarrow \text{ComplainsAbout}(x, y), \text{WorksFor}(x, y), \text{LivesIn}(y, z)$



# Conclusion & Future Work

**Summary:** Semantical and syntactical characterization of OBFs, complexity results, and construction.



# Conclusion & Future Work

**Summary:** Semantical and syntactical characterization of OBFs, complexity results, and construction.

**Future work:**

- ▶ Beyond full CQs without self-joins
- ▶ Less restrictions on messages
- ▶ Alternative notions of optimality